MA5371 – Assignment 5

November 4, 2016

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1. Let ω be a k-form on $\mathbb{R}^n$. Then is it always true that $\omega \wedge \omega = 0$?

2. Let I be an ordered k-tuple. Prove that

$$g^* dx_I = \sum_{\text{increasing } k\text{-tuples } J} \det \left[\frac{\partial g_I}{\partial u_J} \right] du_J$$

   Note: Here g_I denotes $(g_{i_1}, \ldots, g_{i_k})$ and the derivative notation is self-explanatory.

3. Let $U \subset \mathbb{R}^m$ be open and let $g : U \to \mathbb{R}^n$ be smooth. Prove that for any $\omega \in \mathcal{A}(\mathbb{R}^n)$ and $v_1, \ldots, v_k \in \mathbb{R}^m$, we have

$$g^*\omega(a)(v_1, \ldots, v_k) = \omega(g(a))(Dg(a)v_1, \ldots, Dg(a)v_k).$$

4. let $g : (0, \infty) \times (0, \pi) \times (0, 2\pi) : \mathbb{R}^3$ be the usual spherical cooridnates map. Compute $g^*(dx \wedge dy \wedge dz)$.

5. Let C be a smooth closed curve in the plane. Show that

$$\int_C y dx = -\int_C x dy.$$

   Interpret these integrals geometrically.

6. Given an example of a closed 1-form on an open subset of $\mathbb{R}^2$ that is not exact.

7. Use Stoke's theorem to find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1.$$

8. Prove that a $k-$manifold (with or without boundary) M is orientable iff there exists a nowhere vanishing k-form defined on M.

9. Let M be a compact, oriented k-manifold (without boundary) and let ω be a $k-1$-form. Show that

$$\int_M d\omega = 0.$$

   Show by an explicit counter-example that this is not true if M is not compact.