MA5371 – Assignment 3

Sep 23, 2016

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1. Which of the following subsets of $\mathbb{R}^2$ are 1-dimensional manifolds?
   a) The union of the x-axis and y-axis.
   b) The set $\{(x, y) : y = |x|\}$.
   c) The set $\{(x, y) : y \in \mathbb{Q}\}$.
   d) The figure '8' which is the union of the circle of radius 1 centred at $(0, 1)$ and the circle of radius 1 centred at $(0, -1)$.

2. Show that the following sets are manifolds by showing that they satisfy **all** of the three equivalent definitions of a manifold:
   a) $\{(t, t^2, t^4) : t \in \mathbb{R}\}$.
   b) the zero set of the function $f : \mathbb{R}^3 \to \mathbb{R}$ where $f(x, y, z) = (x^2 + y^2 - 1, x^2 + y^2 + z^2 - 2x)$.
   c) the zero set of the function $x^2 + y^2 - z^2$ defined on $\mathbb{R}^3 \setminus \{0\}$

3. (Torus in $\mathbb{R}^3$). Consider the circle in the xz-plane centred at the point $(R, 0, 0)$ of radius $r, r < R/2$. Now rotate this circle about the z-axis. What we get is a donut-type object. Prove that this object is a 2-dimensional manifold.

   Hint: Use polar coordinates to parametrize the circle and then think about how you can parametrize the torus which is obtained by rotating this circle.