MA5360 – Assignment 4
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1. Let f be an entire function and suppose that $f(\mathbb{C})$ misses a disk. What can you say about f?
2. Let f be an entire function and suppose that for some $n \geqslant 0$
$$\lim_{n\to\infty} \frac{f(z)}{z^n}$$
converges to a finite quantity. Prove that f is a polynomial.
3. Let $f : \mathbb{D} \to \mathbb{D}$ be holomorphic. Suppose that for some $z_0 \in \mathbb{D}$ we have $f(z_0) = z_0$ and $f'(z_0) = 1$. What can you say about f?
4. Let $f \in H(\mathbb{D}) \cap \mathcal{C}^0(\overline{\mathbb{D}})$ and suppose that f vanishes on some arc on the unit circle. Show that $f \equiv 0$.
5. Let $f \in D'_\varepsilon(a)$ be such that f takes only values in the upper-half plane. Determine the type of singularity of f at a.
6. Prove that the set of zeroes of a non-identically zero meromorphic function on a domain D is a discrete subset of D.
7. Use the argument principle to given an alternative proof of the local mapping theorem.
8. What type of singularity is the origin for the following functions:
   a) $\frac{1}{\tan z} - \frac{1}{\sin z}$.
   b) $\frac{\sin z}{z^4}$.
   c) $\frac{1}{e^z - 1}$.
9. Let f be a rational function. Show that the sum of all the residues, included at infinity, is equal to 0.
10. Go through all the examples in the textbook in section 5.8.