MA5360 – Assignment 3
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1. Let $\mathbb{D}$ be the unit disk and let $a \in \mathbb{D}$. Write down explicitly the formula of a holomorphic map $f : \mathbb{D} \to \mathbb{D}$ that interchanges 0 and a.
2. Prove that any fractional linear transformation maps a pair of concentric circles onto another pair of concentric circles and the ratio of their radii is constant.
3. Compute $\int_\gamma e^z dz$ where $\gamma(t) = (t, \sin t), t \in [0, \pi]$.
4. Let f be holomorphic in a neighborhood of a closed rectangle R except for finitely many points $z_0, \ldots, z_n \in \text{int}(R)$ and suppose that $\lim_{z \to z_j}(z - z_j)f(z) = 0$. Prove that $\int_R f(z)dz = 0$
5. Compute the integral
$$\int_0^{2\pi} e^{\cos\theta} \sin(n\theta - \sin\theta) d\theta$$
6. Prove that if f is a continuous function on an open convex set U and holomorphic on $U \setminus \{z_0\}, z_0 \in U$, then $\int_\gamma f(z)dz = 0$ for any closed path γ such that $\gamma^* \subset U$.
7. Let γ be a closed path in $\mathbb{C}$ that misses 0. Show directly that the value of
$$\frac{1}{2\pi i} \int_\gamma \frac{dz}{z - z_0}$$
is an integer.
8. Prove that if U is bounded domain with positively oriented piece-wise regular boundary and $f \in \mathcal{C}^0(\overline{U}) \cap H(U)$, then $\int_{\partial U} f(z)dz = 0$.